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Transport in two-fluid magnetohydrodynamic turbulence

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We present a theory of transport of magnetic flux and momentum in two fluid three-dimensional reduced magnetohydrodynamic (MHD) turbulence. By including the effects of shear flows and magnetic fields consistently, we show that kinetic Alfven waves can help weaken the quenching in turbulent transport of a strong magnetic field B_0 found in single fluid MHD turbulence, leading to turbulent magnetic diffusivity $\eta_T \propto (\eta/\Omega)^{1/3} B_0^{-2}$. Here, η and Ω are the Ohmic diffusivity and the shearing rate of the shear flow. Momentum transport is diffusive, with the value of eddy viscosity larger than that in single fluid MHD turbulence. The effects of drift waves are found to be weaker. Implications for the instability of shear flows are discussed.

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The transport process in a variety of systems, including laboratory, astrophysical, and space plasmas, is often observed or inferred to be anomalous, occurring on time scales much shorter than what is expected from molecular processes. Understanding basic physical mechanisms necessary for this fast transport has, however, been elusive, challenging many theoreticians in the past. The main difficulty in theory lies in the identification of mechanisms which can dramatically enhance transport to the values which are far above those based on collisional processes, especially for systems of large size with many degrees of scales or freedom.

As one of the promising mechanisms for enhanced transport, turbulence has been advocated, for instance, to explain magnetic activities occurring on short time scales such as solar cycles and coronal heating in the Sun, galactic dynamos, and major disruption in laboratory plasmas. However, recent numerical and analytical studies have provided convincing evidence that turbulence can lead to fast transport of magnetic fields only in the limit of pathetically weak magnetic fields where the back-reaction of magnetic fields is negligible (see, e.g., [1–3]). In particular, Kim and Diamond have shown that in three-dimensional (3D) reduced magnetohydrodynamic (RMHD) turbulence, turbulent diffusion of large-scale magnetic fields B_0 is significantly slowed down to the rate $\eta_T \propto B_0^{-2}$ [3] as Lorentz force backreaction turns random turbulent eddies into a packet of Alfven waves (the so-called Alfvenization), favoring the equipartition between velocity and magnetic fluctuations. Furthermore, turbulent magnetic diffusion η_T is proportional to Ohmic diffusivity η as $\eta_T \propto \eta$, leading to a slow turbulent reconnection rate $\propto \eta^{1/2}$. The transport can be quenched further by flow shear Ω as the latter quenches turbulence by shear stabilization [4–6]. Kim [7] and Kim and Dubrulle [8] have demonstrated this clearly in 2D MHD where turbulent diffusion of magnetic field is reduced not only by Alfenization but also by flow shear, with the scaling $\eta_T \propto (\eta/\Omega)^{2/3} B_0^{-2}$. The dependence of turbulent magnetic diffusivity on η here is, however, weaker than without flow shear.

Momentum transport is also critically affected by shear flow and magnetic fields [7–9]. In particular, Kim and Dubrulle [8] and Kim *et al.* [9] have predicted that turbulence acting as a source of the generation of shear flows in 2D hydrodynamic (HD) turbulence becomes a sink of shear flows in MHD turbulence, damping the latter for strong magnetic fields as a result of the cancellation of Reynolds stress by Maxwell stress. The tendency of the cancellation of Reynolds stress by Maxwell stress in high β plasmas and the amplification (damping) of shear flow due to Reynolds (Maxwell) stress have been beautifully confirmed by numerical simulations of tokamak core turbulence [10] and edge turbulence [11].

Since aforementioned severe quenching in turbulent transport originates from the fact that magnetic fields are frozen into the fluid in the ideal limit (in the absence of Ohmic diffusion), one plausible way of obtaining a fast transport is to break this frozen-in law, by incorporating two fluid effects of ions and electrons (e.g., [12–15]). In fact, Kleva *et al.* [12] and Cassak *et al.* [15] have shown that kinetic Alfven waves (due to the electron pressure term in the generalized Ohm's law) can lead to fast laminar reconnection, independent of Ohmic diffusivity. A similar fast reconnection was indicated in the case of antiparallel reconnection in electron MHD by the numerical simulation of Cassak *et al.* [14]. In comparison, far much less is understood as to how two-fluid effects affect turbulent transport of magnetic fields, which is critical to understanding transport in high temperature plasmas. Especially, whether it can lead to fast diffusion by compensating a severe quenching found in single fluid MHD turbulence is an outstanding problem.

How the two-fluid effects influence momentum transport is also an important issue. While Kim and Dubrulle [8] and Kim *et al.* [9] have predicted the damping of shear flow as a result of the cancellation of Reynolds stress by Maxwell stress in MHD turbulence, Guzdar *et al.* have suggested that shear flows are amplified in the drift-kinetic-Alfven wave turbulence model [16], by considering the drift-wave branch for modulational instability. In view of the crucial role of shear flows (e.g., zonal flows) in turbulence regulation, in particular, in laboratory plasmas (e.g., see [4–6]), it is important to understand whether two-fluid effects can help the generation of shear flows or not, and if yes, to what extent.

The purpose of this paper is to present a theory of transport of magnetic flux and momentum in two-fluid 3D RMHD turbulence. By a consistent calculation, we show that kinetic Alfven waves can help weaken the quenching in turbulent transport of magnetic fields, leading to a turbulent diffusivity of strong magnetic field $\eta_T \propto \eta^{1/3}$, with a weak dependence on Ohmic diffusivity η. Turbulent momentum transport is found to be diffusive, turbulence acting as a sink of shear flow with positive eddy viscosity. Its value is found to be larger than that in the single fluid MHD turbulence. The effects of drift waves are shown to be weaker than those of kinetic Alfven waves.

The two-fluid MHD equations for electric potential ϕ, magnetic vector potential ψ, and density perturbation n' are as follows (see, e.g., [16]):

$$\frac{d}{dt}\nabla_\perp^2\phi - \mathbf{B}\cdot\nabla\nabla^2\psi = F', \quad (1)$$

$$\frac{d}{dt}\psi + v_*\partial_y\psi - \mathbf{B}\cdot\nabla(\phi - n') = \eta\nabla_\perp^2\psi, \quad (2)$$

$$\frac{d}{dt}n' + v_*\partial_y\phi = \rho_s^2\mathbf{B}\cdot\nabla\nabla_\perp^2\psi. \quad (3)$$

Here, $v_*=\rho_s c_s/L_n$ is the drift velocity due to the gradient of the background density η_0; ρ_s and c_s are ion Larmor radius and sound speed, respectively; $L_n=|\partial_x \ln n_0|$ is the background density gradient length scale; n' is the density perturbation normalized by $n_0/\rho_s c_s$; ψ and B are normalized by background mass density while ϕ by the magnetic field B; d/dt is the total convective time derivative; $\nabla_\perp^2=\partial_{xx}+\partial_{yy}$ is the two-dimensional Laplacian; η is the Ohmic diffusivity; F' is the small-scale forcing on electric potential driving turbulence.

For simplicity, we consider Cartesian coordinates (x,y,z) where x, y, and z represent the local radial, poloidal, and toroidal directions, respectively. We assume that there are a large-scale sheared magnetic field $\langle\mathbf{B}\rangle=B_z\hat{z}+B_0\hat{y}$ and a linear shear flow $\langle\mathbf{U}\rangle=U_0(x)\hat{y}=-x\Omega\hat{y}$ in the y direction, with $\Omega>0$. For simplicity, we further assume a slow variation in z with $k_\parallel=k_z\sim 0$, similar to Kleva *et al.* [12], and consider the reconnection of large-scale magnetic fields $B_0\hat{y}$, in parallel with shear flow $U_0\hat{y}$. In tokamak, $B_0\hat{y}$ represents zonal magnetic field while $U_0\hat{y}$ captures zonal flows or mean poloidal shear flows.

Linear equations for fluctuating variables $\varphi'=\phi',\psi',n',F'$, etc., follow from Eqs. (1)–(3), where a prime denotes fluctuation. In order to solve these equations nonperturbatively for strong shear Ω, we use the time-dependent Fourier transformation for fluctuation φ' as (see, e.g., [7])

$$\varphi'(\mathbf{x},t) = \tilde{\varphi}(\mathbf{k},t)\exp\{i(k_x(t)x + k_y y)\}, \quad (4)$$

with k_x satisfying an eikonal equation

$$\partial_t k_x(t) = k_y\Omega. \quad (5)$$

By using the transformation (4) and the new time variable $\tau=k_x(t)/k_y$ in Eqs. (1)–(3), we can easily obtain

$$\partial_\tau(-k_y^2)[(1+\tau^2)\tilde{\phi}] = -i\gamma k_y^2(1+\tau^2)\tilde{\psi} + \tilde{F}/\Omega, \quad (6)$$

$$[\partial_\tau + \xi(1+\tau^2)]\tilde{\psi} = -i\gamma_*\tilde{\psi} + i\gamma(\tilde{\phi} - \tilde{n}), \quad (7)$$

$$\partial_\tau\tilde{n} = -i\gamma_*\tilde{\phi} - i\gamma(\rho_s^2k_y^2)(1+\tau^2)\tilde{\psi}. \quad (8)$$

Here, $\gamma=k_yB_0/\Omega$, $\gamma_*=k_yv_*/\Omega$, and $\xi=\eta k_y^2/\Omega$. In the following, we focus on the strong shear limit where the shearing effect dominates Ohmic diffusion with $\xi\ll 1$. That is, $\xi\ll 1$ is a small parameter, characterizing the strong shear limit.

To understand the effects of drift waves ($\gamma_*\neq 0$) and kinetic Alfven waves ($\rho_s\neq 0$), it is instructive to solve Eqs. (6)–(8) in the limit of $\gamma_*=0$ and $\rho_s=0$, separately. First, in the case of $\rho_s=0$, one can obtain the following equation for $\tilde{\psi}$:

$$\partial_\tau\{(1+\tau^2)[\partial_\tau + \xi(1+\tau^2)]\tilde{\psi} + \Delta\} + \gamma^2(1+\tau^2)\tilde{\psi} = \frac{\gamma}{ik_y^2\Omega}\tilde{F}, \quad (9)$$

where $\Delta=-i\gamma_*\xi\int d\tau_1 e^{-i\gamma_*(\tau-\tau_1)}(1+\tau_1^2)\tilde{\psi}(\tau_1)$. The solution to Eq. (9) can be found in the limit of strong magnetic field with $\gamma^2=k_y^2B_0^2/\Omega^2\gg 1$ in the form

$$\tilde{\psi}(\mathbf{k},\tau) \simeq \frac{i}{\sqrt{1+\tau^2}}\int^\tau \frac{d\tau_1}{\Omega}\frac{1}{\sqrt{1+\tau_1^2}}\sin\{|\gamma|(\tau-\tau_1)\} \times e^{-\xi Q(\tau,\tau_1)/2}\frac{\tilde{F}(\mathbf{k}(\tau_1),\tau_1)}{k_y^2}, \quad (10)$$

to second order in $O(\gamma^{-1})$. Here, $Q(\tau,\tau_1)=(\tau+\tau^3/3)-(\tau_1+\tau_1^3/3)$. Note that the limit of a strong magnetic field $\gamma>1$ here is valid when the Alfven frequency of a mode (B_0k_y) is larger than the shearing rate Ω. Note that the solution (10) is similar to that found in 2D MHD [7]. On the other hand, the solutions to $\tilde{\phi}$ and $\tilde{n}$ follow from Eqs. (6)–(8) as

$$\tilde{\phi} = -\frac{i}{\gamma}\partial_\tau\tilde{\psi}, \quad (11)$$

$$\tilde{n} = -\frac{\gamma_*}{\gamma}\tilde{\psi}. \quad (12)$$

The transport of momentum and magnetic flux are determined by the correlations between two fluctuations which contribute to the evolution of mean fields as eddy viscosity ν_T via total stress $\langle u_x'u_y' - b_x'b_y'\rangle = -\nu_T\partial_x U_0 = \nu_T\Omega$ and turbulent magnetic diffusivity η_T via magnetic flux $\langle b_x'(\phi'-n')\rangle = -\eta_T\partial_x\psi_0 = \eta_T B_0$, respectively. In order to simplify analytical analysis in computing these, we assume that the statistics of the forcing is homogeneous and stationary with a short correlation time τ_f:

$$\langle\tilde{F}(\mathbf{k}_1,t_1)\tilde{F}(\mathbf{k}_2,t_2)\rangle = \tau_f(2\pi)^2\Pi(\mathbf{k}_1,t_1)\delta(t_1-t_2), \quad (13)$$

where Π is the power spectrum of the forcing F'. A long but straightforward algebra by using Eqs. (10)–(13) then gives us

$$\eta_T \sim \frac{\chi\tau_f}{B_0^2}\int\frac{d^2k}{(2\pi)^2}\frac{\Pi(\mathbf{k})}{k_y^4}\xi^{2/3}, \quad (14)$$

$$\nu_T \sim \frac{\tau_f}{4B_0^2}\int \frac{d^2k}{(2\pi)^2}\frac{\Pi(\mathbf{k})}{k_y^4}. \quad (15)$$

Here, $\xi = \eta k_y^2/\Omega \ll 1$, and $\chi = \Gamma(1/3)3^{-2/3}/2$ is a numerical constant. Interestingly, these results (14) and (15) are exactly the same as those in 2D MHD turbulence [7]. That is, drift waves have no effect on turbulent transport driven by a temporally short correlated forcing. This is basically because turbulence decorrelates too rapidly to be influenced by drift waves as the forcing changes too quickly.

We now show that kinetic Alfven waves due to the electron pressure term have a more interesting effect on turbulent transport. This is mainly because shearing by flow shear rapidly generates small scales in time as $k_x \sim k_y \Omega t$, consequently enhancing ion inertia $\propto \rho_s k \sim \rho_s k_y \Omega t$. Here, k is the characteristic wave number of small-scale turbulence. The enhanced ion inertia effectively makes fluctuations in turbulent velocity weaker than those in magnetic field. As a result, Maxwell stress becomes much larger than Reynolds stress, resulting in a positive eddy viscosity with the value much larger than that in single fluid MHD turbulence.

For the brevity of presentation, only a few main steps leading to these results are now provided. By taking $\gamma_* = v_* = 0$ in Eqs. (6)–(8), one can derive an equation for $\tilde{\psi}$ as

$$\partial_\tau\{S(\tau)[\partial_\tau + \xi(1+\tau^2)]\tilde{\psi}\} + \gamma^2(1+\tau^2)\tilde{\psi} = \frac{\gamma}{ik_y^2\Omega}\tilde{F}_T. \quad (16)$$

Here, $S(\tau) = (1+\tau^2)/[1+\rho_s^2k_y^2(1+\tau^2)]$; $\tilde{F}_T = \tilde{F} - \rho_s^2k_y^2\partial_\tau[S(\tau)\int^\tau d\tau_1 \tilde{F}(\tau_1)]$. We can find a solution to Eq. (16) for large $|\gamma| = |B_0k_y/\Omega|$ to second order in $O(\gamma^{-1})$ as follows:

$$\tilde{\psi}(\mathbf{k},\tau) \simeq \frac{|k_y|}{ik_y^3}H(\tau)\int^\tau \frac{d\tau_1}{\Omega}H(\tau_1) \times \sin\{\Gamma(\tau,\tau_1)\}e^{-\xi Q(\tau,\tau_1)/2}\tilde{F}_T(\mathbf{k}(\tau_1),\tau_1). \quad (17)$$

Here, again, $Q(\tau,\tau_1) = (\tau + \tau^3/3) - (\tau_1 + \tau_1^3/3)$; $H(\tau) = [1+\rho_s^2k_y^2(1+\tau^2)]^{1/4}/\sqrt{1+\tau^2}$; $\Gamma(\tau,\tau_1) = |\gamma|\int_{\tau_1}^{\tau} d\tau_2[1+\rho_s^2k_y^2(1+\tau_2^2)]^{1/2}$. In order to compute η_T and ν_T, we again assume that the forcing F' is homogeneous and stationary with a short correlation time τ_f and has a power spectrum Π [see Eq. (13)]. A long but straightforward algebra then gives us the final results:

$$\eta_T \sim \beta\frac{\tau_f}{B_0^2}\int \frac{d^2k}{(2\pi)^2}\frac{\Pi(\mathbf{k})}{k_y^4}\frac{|\rho_s k_y|}{(1+\rho_s^2k_y^2)^{3/2}}\xi^{1/3}, \quad (18)$$

$$\nu_T \sim \alpha\frac{\tau_f}{\Omega^2}\int \frac{d^2k}{(2\pi)^2}\frac{\Pi(\mathbf{k})}{k_y^2}\frac{|\rho_s k_y \ln \xi|}{(1+\rho_s^2k_y^2)^{3/2}}\xi^{-1/3}, \quad (19)$$

where $\xi = \eta k_y^2/\Omega$; $\alpha = 3^{1/3}\Gamma(4/3)/6$ and $\beta = 3^{2/3}\Gamma(2/3)/6$ are numerical constants. Equations (18) and (19) indicate that turbulent transport of momentum and magnetic flux are quenched by flow shear and magnetic field. Specifically, Eq. (18) shows that $\eta_T \propto B_0^{-2}(\eta/\Omega)^{1/3}$ is reduced due to both magnetic field B_0 and flow shear Ω, with its value decreasing $\propto \eta^{1/3}$ as η becomes small. This dependence of $\eta_T \propto \eta^{1/3}$ with 1/3 power is weak, compared to the single fluid MHD result $\propto \eta^{2/3}$ [8]. The resulting turbulent reconnection rate of large-scale magnetic fields is expected to scale $\propto \eta_T^{1/2} \propto \eta^{1/6}$ (e.g., see [3]), depending very weakly on Ohmic diffusivity. The faster reconnection rate compared to the single fluid MHD turbulence here results from a faster formation of small scale structures (or, cascade of magnetic energy) as the electron pressure term breaks the frozen-in condition of magnetic fields. We note that our result here is not in line with Cassak *et al.* [15], who obtained a fast reconnection for sufficiently small η although a direct comparison could be misleading as the result in Cassak *et al.* [15] is more relevant to laminar reconnection. The use of hyperdiffusivity in Cassak *et al.* [14,15] might, however, be questionable as it could alone lead to magnetic dissipation and reconnection.

On the other hand, Eq. (19) demonstrates that momentum transport is mainly quenched by flow shear with a positive eddy viscosity $\nu_T \sim (\rho_s k_y)^{-2}\Omega^{-5/3}|\ln \Omega|$ for $\rho_s k_y > 1$. It is interesting to note that for reasonable parameter values $\gamma^2 > 1$, ν_T in Eq. (19) is larger than $\nu_T \propto B_0^{-2}$ in the single fluid MHD case [8,9]. This is because ion inertia, enhanced by flow shear, causes velocity fluctuations much weaker than magnetic fluctuations, as noted previously.

We emphasize that the positive eddy viscosity (15) and (19) indicates that for strong magnetic field (i.e., $|B_0k_y|/\Omega \gg 1$), a large-scale shear flow damps due to turbulence, rather than being amplified. However, when the strength of magnetic field is sufficiently weak, eddy viscosity becomes negative (see also [10,11]). In order to demonstrate this point, it is instructive to consider the kinematic limit where the backreaction of magnetic field onto the fluid and particles is neglected, i.e., in the limit where the terms $\propto \gamma\tilde{\psi}$ in Eqs. (6) and (8) are neglected. By taking these terms to be zero and after a long but straightforward algebra, one can show that $\nu_T \propto -\Omega^{-2}$ and $\eta_T \propto \Omega^{-2}$. These results, which are similar to those in the 2D HD case [8], demonstrate that a shear flow is amplified with a negative viscosity while the transport of magnetic flux is quenched due to flow shear only. This is consistent with an instability (growth) of shear flow in Guzdar *et al.* [16] since the kinematic limit is equivalent to taking drift wave branch for modulational instability. Note, however, that in our quasi-2D model with $k_\parallel = 0$, a large-scale magnetic field cannot grow.

In summary, we have shown that in two fluid 3D RMHD turbulence, kinetic Alfven waves have an interesting effect on turbulent transport of magnetic flux and momentum due to the ion inertia which is enhanced by flow shear. In particular, the dissipation rate of a large-scale magnetic field is faster compared to the case of single fluid MHD turbulence; its value depends weakly on Ohmic diffusivity as $\eta^{1/3}$, likely to give turbulent reconnection rate $\propto \eta^{1/6}$. On the other hand, turbulent momentum transport is found to be diffusive, with the value of eddy viscosity larger than that for single fluid MHD turbulence. It would be worth performing numerical computations in order to verify these predictions. In particular, the verification of weak dependence on η of the turbulent reconnection rate could be a challenge in numerical computations. It would also be interesting to generalize our model to incorporate a finite correlation time of the forcing and to study the generation of large-scale magnetic field (e.g., zonal field) in three dimensions [16] and the interplay between shear flow and zonal field. Of particular interest would be a self-consistent dynamics of tearing modes [17], by including the effect of the gradient (magnetic shear) of reconnecting magnetic field (e.g., see [18]). Other two-fluid effects, such as the $\mathbf{J}\times\mathbf{B}$ Hall effect and/or kinetic effects, could contribute to fast turbulent transport and should be investigated. These issues will be addressed in future publications.

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